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The composite functional statistical criterion for evaluating the effectiveness of the automatic control process and system

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Abstract. The process of managing and controlling equipment is a broader concept than data transmission. It includes collecting the information about the controlled parameters, transferring the information, its processing, the development of control actions and their transfer to the object in order to bring the controlled parameters back to normal. These tasks are solved by the control and monitoring system. Modern ACMS have not only a larger number of elements, but also a more complex internal structure. Even if all elements of ACMS function are in a reliable way in the process of its operation, the performance of the task assigned to the system cannot be treated as a reliable event. One of the essential features of ACMS is that they are meant to improve the quality of the controlled objects. To quantify the quality of complex systems, effectiveness indicators are used. Indicators of the quality of the system performance of the assigned functions are called the effectiveness of system operation or simply effectiveness. The main idea of evaluating the effectiveness of ACMS is to get the opportunity to compare systems that are identical in purpose, even if they differ from each other in the principles of design and operation. There are certain connections between the individual elements of ACMS, the system as a whole and the controlled object through which they interact with each other. From the point of view of evaluating ACMS effectiveness, the information content of these links is crucial, since the processes of automatic control, for example, in the field of electrical energy production and distribution, are carried out on the basis of information received about the controlled object.

1. Introduction

Analysis of achievements in the field of automatic control and monitoring systems shows that modern manufacturers pay attention to the strength of the products offered and characterize them according to the criteria:

- calculated multiplication of the system accuracy indicator by the rate of its operation;
- by signal volume;
- by the probability of erroneous message reception.

And the average consumer has to use these parameters when choosing complex and expensive products.

Currently, there is no rigorous mathematical substantiation of a generalized criterion for assessing the effectiveness of a process and an automatic control and monitoring system. Which should characterize the information capacity of the process and the automatic control and monitoring system.

Thus, the article raises the topical issue of the mathematical substantiation of evaluating the effectiveness of the control and management process carried out by the ACS through the introduction of the entropy of the controlled object, taking into account the entropy of the ACS, caused by errors in the implementation of the monitoring and control process of each subsystem of the object.

2. The rationalization technique for a composite cross-section

2.1 Problem statement

Modern automatic checkout and control systems are gaining wider and wider popularity in various areas of the national economy in general and in transport sphere in particular. At the same time, we require definite criteria to generally evaluate the effectiveness of the process and automatic control and monitoring systems (ACMS) themselves.

2.2 Analysis of the recent researches and publications

The problem has been analyzed in detail in [1-13]. The scientists have not suggested a profound solution to the task under study. There have been developed some criteria to evaluate the effectiveness of the automatic control and monitoring process. However, there are several of them, which causes certain difficulties in evaluating the effectiveness of the process and ACMS.

2.3 The aim of the article

The article is aimed at selecting and justifying a composite criterion to evaluate the effectiveness of the process and automatic control and monitoring system.

2.4 The main body

A composite criterion for the evaluation of the effectiveness has to characterize the information capacity of the process and the system of automatic control and monitoring.

Modern automatic control and monitoring systems use discrete messages based on the fact that arbitrarily complex messages can be transmitted using a sequence of binary digits 0 and 1.

A binary sequence can be used to transmit an event message selected from $N=2^m$ possible events, where $m = \log_2 N$ is the length of the sequence (the number of bits in a binary number). Thus, the length of the sequence required to transmit a certain message is proportional to the logarithm of the number of possible messages. It is obvious that the maximum amount of information $H_{\max}$, contained in a message is proportional to its length, i.e.

$$H_{\max} = \log_2 N . \quad (1)$$

Value $H_{\max}$ indicates an upper bound on the amount of information that can be contained in a message.

The actual amount of information depends not only on the number of possible messages, but also on their probabilities, since messages are chosen from a set of possible non-uniform messages. On the grounds of the mentioned above, the concept of a quantitative measure of information is introduced.

$$H(X) = -\sum_{i=1}^{N} P(X_i)\log_2 P(X_i), \quad (2)$$

where $P(X_i)$ is a probability of the i-message from the set N.

In the case when the probabilities of all messages, except one, turn to zero, i.e. if $P(X_i) = 0$ and if $P(X_k) = 1\ (k \neq i)$, the amount of information is zero. If it is known in advance that all possible messages in the set *N* are equally probable, the amount of information is maximum.

Thus, in all cases, except limiting ones ($N_{\min} = 0$ and $H_{\max} = \log_2 N$), the inequality holds true

$$0 < H < \log_2 N \text{ .(3)} \quad (3)$$

Value *H* is called the entropy of the random value *X*. It is a measure of the initial uncertainty of the result when selecting messages *X* from the set of possible messages. Message selection completely removes this uncertainty and delivers an amount of information equal to *H*.

The amount of information received by such a system in the process of monitoring and controlling an object over a time interval (t, τ) is equal to the change in the amount of entropy:

$$I(X,t,\tau) = H_0(X,t,\tau) - H_k(X,t,\tau), \quad (4)$$

where $H_0(X,t,\tau)$ is the entropy, which describes the uncertainty of the object under control and ACMS before monitoring and controlling (X is the set of object states at the time point t, corresponding to the moment when monitoring is stopped, τ is the time point corresponding to the moment when control system stops operating; $H_k(X,t,\tau)$ is the conditional entropy of the controlled object and ACMS in the implementation of the monitoring and control process.
Each state of the controlled object at any time t is characterized by the probability of completing the task in the time interval (t, τ):

$$P(t,\tau) = P_{BO}(t,\tau) \cdot P_{ПО}(t,\tau), \quad (5)$$

where $P_{BO}(t,\tau)$, $P_{ПО}(t,\tau)$ are respectively the probabilities of the absence of sudden and gradual failures of the controlled object and the control and monitoring system.

If sudden and gradual failures are independent, the entropy of the control and monitoring process is the sum of the entropies caused by these failures:

$$H_0(X,t,\tau) = H_{BO}(X,t,\tau) + H_{ПО}(X,t,\tau), \quad (6)$$

where $H_{BO}(X,t,\tau) = -\{P_{BO}(X,t,\tau)\log_2 P_{BO}(X,t,\tau) + (1 - P_{BO}(X,t,\tau))\log_2(1 - P_{BO}(X,t,\tau))\}$ in accordance with the expression (2);

$H_{ПО}(X,t,\tau) = -\int_{-\infty}^{\infty}\cdots\int f_{ПО}(X,t,\tau)\log_2 f_{ПО}(X,t,\tau)dX$ is the entropy caused by the influence of the gradual failures on the output parameters of the control and monitoring process (here $f_{ПО}(X,t,\tau)$ is the distribution density of the output parameter X, subjected to the influence of gradual failures).

The optimal level of the controlled object uncertainty is selected from the condition ensuring a given probability of the task completion by the object:

$$\begin{aligned} H_{ОПТ}(X,t,\tau) = -\{&P(X,t_{kk},\tau_{kPO})\log_2 P(X,t_{kk},\tau_{kPO}) + \\ &+ [1 - P(X,t_{kk},\tau_{kPO})]\log_2[1 - P(X,t_{kk},\tau_{kPO})]\}, \end{aligned} \quad (7)$$

where $P(X,t_{kk},\tau_{kPO})$ is the probability that the object as well as control and monitoring system will complete the task completion in the time interval (t_{kk},τ_{kPO}), which starts at the time moment when monitoring stops t_{kk} (monitoring end), and stops at the time moment when the controlled object stops operating τ_{kPO} (end of the object operation).

The control time τ_k should be kept to a minimum so that the maximum amount of information with the greatest reliability can be obtained per unit of time. At the same time, this minimum should

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not exceed the permissible time for taking the object out of the readiness state, as this will lead to an unjustified complication of the control and monitoring system.

The composite criterion of effectiveness should characterize the information capacity of the process and the automatic control and monitoring system.

The amount of information received by the ACMS in the process of monitoring and controlling an object over a time interval is determined in accordance with the dependence:

$$I(t,\tau) = H_0(t,\tau) - H(t,\tau), \tag{8}$$

where $H_0(t,\tau)$ is the entropy that describes the uncertainty of the controlled object and ACMS before control and monitoring starts. It is determined by the formula (6);
$H(t,\tau)$ is the remaining entropy of the controlled object and ACMS after the process of object controlling and monitoring is completed. It is determined by the formula (7).

The equality (8) describes the real information capacity of ACMS. The potential possibility of the system is the value

$$I_{\Pi}(t,\tau) = H(t,\tau). \tag{9}$$

The effectiveness of ACMS from the informational point of view can be evaluated by the criterion

$$Э_I(t,\tau) = \frac{I_P(t,\tau)}{I_{\Pi}(t,\tau)}, \tag{10}$$

where $I_P(t,\tau) = I(t,\tau)$ is the real amount of information calculated according to (8).

Taking into account (8) and (9), we get expression (11) to evaluate the effectiveness of ACMS

$$Э(t,\tau) = \frac{H_0(t,\tau) - H(t,\tau)}{H_0(t,\tau)} \tag{11}$$

According to the expression (11) it is easy to notice that the ideal ACMS will have the effectiveness value $Э_I(t,\tau) = 1$, and the real one – the value $Э_I(t,\tau) < 1$. However, the criterion (11) does not take into account the dynamics of the control and monitoring process, complexity and cost of ACMS. These disadvantages can be eliminated by introducing a different ratio for ACMS effectiveness

$$Э(t,\tau) = \frac{K_I(t,\tau)}{K_{IO}(t,\tau)}, \tag{12}$$

where $K_I(t,\tau) = \dfrac{I_{\max}(t,\tau)}{C_{\Sigma}(t,\tau)}$ is the maximum amount of information obtained by ACMS per unit of its total cost;

$I_{\max}(t,\tau) = \sum_{i=1}^{m} I_{i\max}(t,\tau)$ is the maximum amount of information obtained when monitoring the process by *m* parameters;

$K_{IO}(t,\tau) = \dfrac{I_{\max\max}(t,\tau)}{C_{\min}(t,\tau)}$ is the maximum average amount of information received by ACMS when controlling the process by *m* parameters per unit of the minimum total cost of the idealized ACMS;

$I_{\max\max}(t,\tau) = \sum_{i=1}^{m} I_{i\max\max}(t,\tau)$ is the maximum average amount of information obtained when the object is controlled by *m* parameters.

Taking into account the abovementioned relations (8), (9), (12) and the equality $I_{\max\max}(t,\tau) = m$ we get the final formula to evaluate the effectiveness ACMS:

$$Э(t,\tau) = \frac{\sum_{i=1}^{m}[H_{0i}(t,\tau) - H_i(t,\tau)]C_{\min}(t,\tau)}{m\sum_{i=1}^{m}C_i(t,\tau)}. \tag{13}$$

3. Conclusion

Thus, to evaluate the effectiveness of the monitoring and control process carried out by ACMS, it is necessary:
- to determine the entropy of each subsystem of the object and ACMS before the control process starts;
- to determine the entropy of the controlled object, taking into account the entropy ACMS, caused by the errors taking place during the process of monitoring and control of each subsystem of the object;
- to calculate the initial cost $C_{\min}(t,\tau)$ and the final cost C of ACMS;
- to make calculations using the formula (13).

It is worth mentioning that the variation range of the composite statistical criterion corresponds to condition $0 \le Э(t,\tau) \le 1$.

Advanced ACMS with high efficiency have a composite index of effectiveness close to one.

References

[1] O.Fomin, A. Akimov, Ju Akimova, A. Mastepan. The criteria choice of evaluating the effectiveness of the process and automatic control systems.//Procedia Computer Science, 2019, Vol. 149, p.246-251.
[2] Avduevskii V. Reliability and effectiveness in engineering. Volume 1. Methodology. Organization. Terminology / V. Avduevskii, I. Appolonov, E. Barzilovich et. al. Ed. A. Rembeza. – Moscow: Mashinostroenie, 1986, 224 p.
[3] Papkov B. Reliability and effectiveness of power supply / B. Papkov, D. Pashali // Ufa: Ufa Sate Aviation Technical University, 2005, 380 p.
[4] Muginshtein L.A. Power optimal traction calculation for operation of trans of increased mass and length [Text] / L.A. Muginshtein, L.A. Yabko // IHHA, June 22-55, Shanghai, China, 2009.
[5] Fokin J. Reliabilty and effectiveness of electrical networks. – Moscow: Vysshaia shkola, 1989, 149 p.
[6] SEL-421 Reference Manual, p. R.1.109. Available at http://www.seline.com/instructionmanual.htm.
[7] M.Gugerty,R.Jenkins, and D. Dulezilek, "Case Study Comparison of Serial and Ethernet Digital Communications Tehnologies for Transfer of Relay Quantities", proceeding of the Distribu TECH Conference and Exhibition, San Diego, CA, February 2007.
[8] Liamets Y., Podchivaline A., Chevelev A., Nudelman G., Zakonjsek J. Informational task of relay protection// CIGRE SC BS Colloquium, Sydney, Australia, 2003. Paper 213.
[9] J.Lewis Blackburn, Tomas J. Domin/ Protective Relaying Principles and Applications / 2007.
[10] Prutik A., Shmojlov A. Algorithm for evaluating technical effectiveness of relay protection and automation. Scientific problems of transport in Siberia and the Far East, №1, 2009, p. 192-195.
[11] Tadjibaev A. Methods and tools for evaluating the state of power equipment / A. Tadjibaev, F. Khalilov, G. Smirnov. Issue 8. – Saint Petersburgh, 1999,174 p.
[12] Matveev V. Evaluation of the efficiency of the power supply system in Topka / V. Matveev, M.

Eremeev, M. Vasenin // Bulletin of the Kuzbass State Technical University, 2010, №6, p. 74-78.

[13] Akimov O. Evaluating the effectiveness of power supply systems on electrified railways / O. Akimov, D. Kukushkin, A. Popadin, A. Pryschepa // Systems of information processing, coll. of scien. papers Kharkiv Air Force University, 2016, Issue 7(114), p.12-14.